

A Joint Marginal-Conditional Model for Multivariate Longitudinal Data

Supplemental Materials

James Proudfoot, Walter Faig, Loki Natarajan and Ronghui Xu

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1 Scaling of the parameters

Most of the parameters in $\hat{\theta}$ need to be transformed back to their original scale for applications. Given two random variable X and Y , the expectation and variance of their product is given by

$$\text{E}[XY] = \text{E}[X]\text{E}[Y] + \text{Cov}[X, Y] \quad (1)$$

$$\text{Var}[XY] = \text{E}[X]^2\text{Var}[Y] + \text{E}[Y]^2\text{Var}[X] + 2\text{E}[X]\text{E}[Y]\text{Cov}[X, Y] + \quad (2)$$

$$\text{Var}[X]\text{Var}[Y] + \text{Cov}[X, Y]^2 \quad (3)$$

The elements of the above equations are readily available from the numerical approximation for $\text{Var}[\hat{\theta}]$ when transforming $\tilde{\beta}$ back to β (via $\tilde{\beta} \times \hat{\sigma}$ for the appropriate $\hat{\sigma}$ corresponding to the same outcome). However, the elements of \tilde{D} need to be transformed back by a factor of $\hat{\sigma}^2$, requiring terms that are not readily available in the matrix $\text{Var}[\hat{\theta}]$ (such as $\text{Cov}[\tilde{D}, \hat{\sigma}^2]$). To amend this, we re-parameterize the elements of $\hat{\theta}$ corresponding to the random effects covariance matrix \tilde{D} as standard deviations (i.e. random intercept standard deviation rather than variance), and replace the covariance elements of $\hat{\theta}$ corresponding to \tilde{D} with correlations

(i.e. correlation between random intercept and slope rather than covariance). This eliminates the need to scale by anything but $\hat{\sigma}$, as correlations do not need to be transformed and the standard deviation of the random effects are transformed by $\hat{\sigma}$.

2 Additional simulation results

Figure S1 and Figure S2 are density plots of the parameter estimates and their standard errors (SE) corresponding to the simulation results from the first half of Table 1 (joint models) in the main article. Figure S3 is a boxplot of the SE for $\sqrt{D_1}$.

Table S1, Table S2, Table S3 and Table S4 contain results from fitting each outcome separately using ‘*lme4*’ that correspond to Tables 2 and 4 in the main paper, for error distributions that are mixture of normals or Weibull.

Results from 5,000 simulation runs ($N = 100$) with three correlated outcomes are given in Table S5 and Table ?? for normally distributed error, and Table S6 and Table ?? for mixture of normals error. A similar simulation framework as that of the main article was followed with the true parameter values given in the tables and

$$R = \begin{pmatrix} 1.0 & 0.5 & 0.1 \\ 0.5 & 1.0 & 0.3 \\ 0.1 & 0.3 & 1.0 \end{pmatrix}.$$

While the true correlation was unstructured, independent and exchangeable working correlation were also fitted to the simulated data.

Results from 5,000 simulation runs with a larger sample size $N = 1,000$ are in Table S7, Table S8, Table S9, and Table S10 below.

3 Additional model fits to WHEL data

Table S14, Table S15 and Table S13 contain separate model fit results to the WHEL data.

Table S14 contains intervention by time interaction, and Table S15 fits both intervention groups and categorical time points.

Figure S1: Density of parameter estimates from the 5000 simulation runs corresponding to the top half of Table 1 (joint models) in the main article. The red dashed lines represent the empirical mean, while the blue dotted line represents the median.

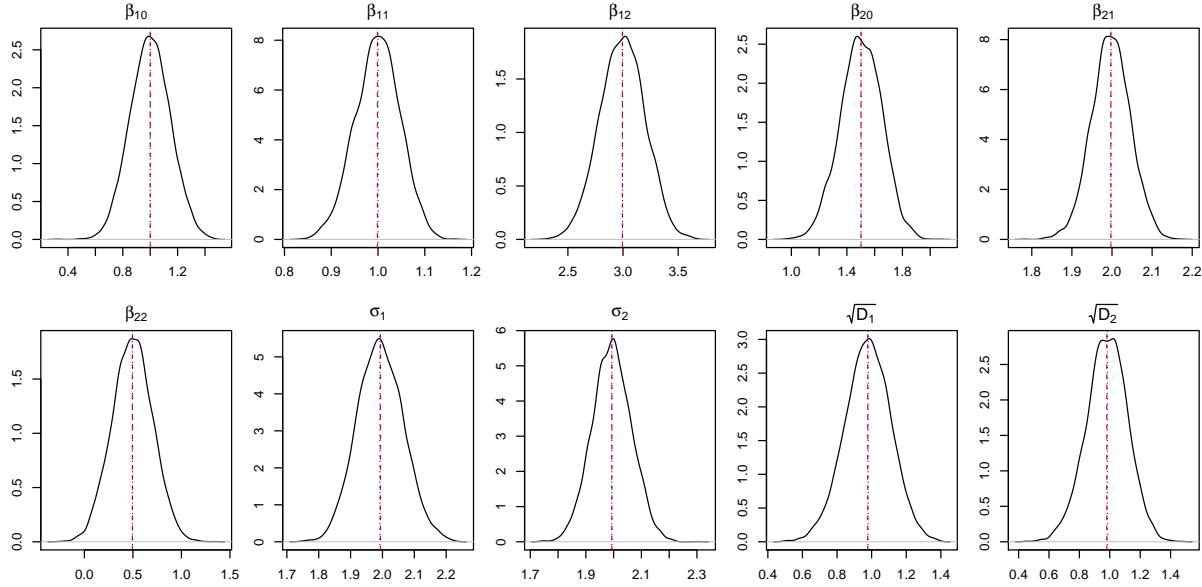


Figure S2: Density of estimated standard error (SE) of each parameter from the 5000 simulation runs corresponding to the top half of Table 1 (joint models) in the main article. The red dashed lines represent the empirical mean, while the blue dotted line represents the median.

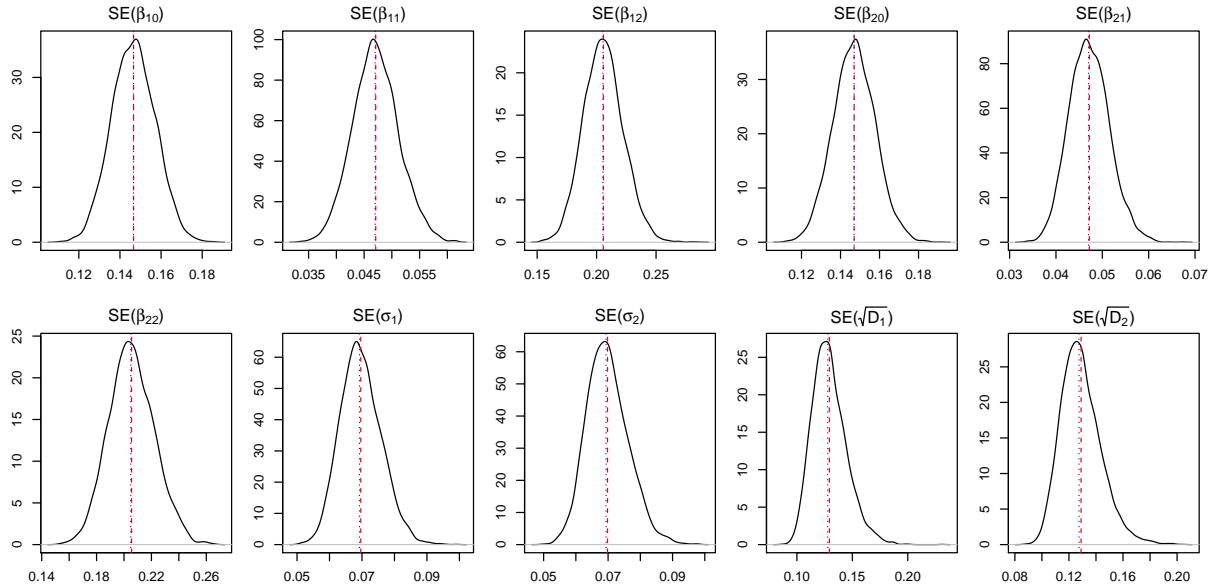


Figure S3: Boxplot of SE for $\sqrt{D_1}$ from the 5000 simulation runs corresponding to the top half of Table 1 (joint models) in the main article. The red dashed line represent the sample SD.

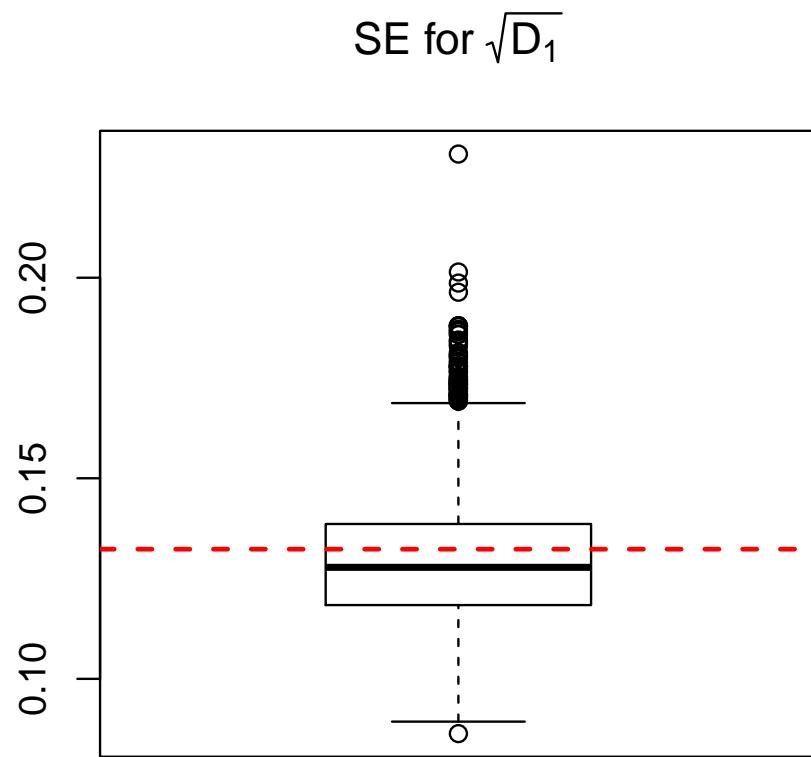


Table S1: Simulation results for two correlated outcomes, with a random intercept on each outcome. The errors were distributed as mixture of normals.

Work Corr.	True	Joint Model				Separate Models			
		Estimate	SE	Sample SD	Coverage	Estimate	SE	Sample SD	Coverage
Ind.									
β_{10}	1	1.000	0.147	0.149	94.0%	1.001	0.148	0.149	94.4%
β_{11}	1	0.999	0.047	0.048	94.4%	1.001	0.047	0.048	94.9%
β_{12}	3	2.996	0.206	0.209	94.4%	3.000	0.207	0.209	94.6%
β_{20}	1.5	1.500	0.147	0.148	94.8%	1.502	0.148	0.149	94.8%
β_{21}	2	1.997	0.047	0.047	94.5%	1.999	0.048	0.047	94.7%
β_{22}	0.5	0.495	0.206	0.205	94.8%	0.496	0.207	0.206	95.1%
σ_1	2	1.995	0.068	0.070	94.1%	1.999		0.070	
σ_2	2	1.996	0.068	0.070	93.9%	2.001		0.070	
$\sqrt{D_1}$	1	0.978	0.129	0.133	93.9%	0.989		0.133	
$\sqrt{D_2}$	1	0.982	0.130	0.132	94.3%	0.993		0.132	
Exch.									
β_{10}	1	1.000	0.147	0.149	93.8%				
β_{11}	1	0.999	0.047	0.048	94.5%				
β_{12}	3	2.996	0.205	0.209	94.6%				
β_{20}	1.5	1.500	0.147	0.148	94.7%				
β_{21}	2	1.997	0.047	0.047	94.4%				
β_{22}	0.5	0.495	0.205	0.206	94.9%				
σ_1	2	1.993	0.069	0.069	94.1%				
σ_2	2	1.995	0.068	0.069	94.5%				
$\sqrt{D_1}$	1	0.982	0.126	0.129	94.1%				
$\sqrt{D_2}$	1	0.985	0.127	0.129	94.4%				
R	0.3	0.319	—	—	—				

Table S2: Simulation results for two correlated outcomes, with a random intercept on each outcome. The errors were distributed as Weibull.

Work Corr.	True	Joint Model				Separate Models			
		Estimate	SE	Sample SD	Coverage	Estimate	SE	Sample SD	Coverage
Ind.									
β_{10}	1	0.997	0.147	0.146	94.6%	0.999	0.147	0.148	94.8%
β_{11}	1	0.989	0.050	0.049	95.2%	0.999	0.047	0.048	95.0%
β_{12}	3	2.968	0.210	0.208	94.1%	3.000	0.206	0.203	95.3%
β_{20}	1.5	1.496	0.149	0.145	94.9%	1.502	0.148	0.147	94.9%
β_{21}	2	1.980	0.056	0.049	96.8%	2.000	0.047	0.047	95.4%
β_{22}	0.5	0.491	0.205	0.204	94.3%	0.503	0.207	0.212	94.6%
σ_1	2	1.983	0.197	0.225	88.2%	1.983		0.222	
σ_2	2	1.987	0.198	0.224	88.4%	1.993		0.220	
D_1	1	0.972	0.130	0.134	94.4%	0.992		0.136	
D_2	1	0.974	0.130	0.134	95.0%	0.993		0.135	
Exch.									
β_{10}	1	0.997	0.147	0.146	94.6%				
β_{11}	1	0.989	0.049	0.048	95.2%				
β_{12}	3	2.968	0.210	0.208	94.4%				
β_{20}	1.5	1.496	0.149	0.145	94.9%				
β_{21}	2	1.980	0.056	0.049	96.8%				
β_{22}	0.5	0.491	0.205	0.204	94.1%				
σ_1	2	1.982	0.198	0.225	88.2%				
σ_2	2	1.986	0.198	0.224	88.4%				
D_1	1	0.974	0.127	0.131	94.8%				
D_2	1	0.976	0.127	0.131	95.0%				
R	0.3	0.313	—	—	—				

Table S3: Simulation results for two correlated outcomes, with a random intercept and a random slope on each outcome. The errors were distributed as mixture of normals.

Work Corr.	True	Joint Model				Separate Models			
		Estimate	SE	Sample SD	Coverage	Estimate	SE	Sample SD	Coverage
Ind.									
β_{10}	1	0.997	0.155	0.154	95.0%	0.998	0.155	0.154	94.9%
β_{11}	1	0.997	0.114	0.115	94.7%	0.999	0.114	0.116	94.6%
β_{12}	3	2.993	0.222	0.223	94.4%	2.997	0.223	0.224	94.5%
β_{20}	1.5	1.493	0.307	0.309	94.7%	1.495	0.307	0.310	94.5%
β_{21}	2	1.998	0.184	0.186	94.5%	2.001	0.184	0.186	94.5%
β_{22}	0.5	0.500	0.439	0.441	94.6%	0.500	0.441	0.441	95.2%
σ_1	2	1.996	0.077	0.079	94.2%	1.999		0.080	
σ_2	4	3.989	0.157	0.159	93.6%	3.998		0.163	
$\sqrt{D_{11}}$	1	0.982	0.142	0.143	94.7%	0.993		0.151	
$\sqrt{D_{12}}$	1	0.989	0.090	0.092	93.0%	0.998		0.093	
ρ_1	0.5	0.512	0.137	0.143	93.0%	0.508		0.144	
$\sqrt{D_{21}}$	2	1.962	0.286	0.282	95.1%	1.968		0.340	
$\sqrt{D_{22}}$	1.5	1.480	0.158	0.162	93.0%	1.494		0.162	
ρ_2	0.5	0.511	0.148	0.158	91.7%	0.503		0.163	
Exch.									
β_{10}	1	0.996	0.154	0.153	95.0%				
β_{11}	1	0.998	0.114	0.115	94.7%				
β_{12}	3	2.993	0.220	0.221	94.5%				
β_{20}	1.5	1.492	0.306	0.309	94.5%				
β_{21}	2	1.998	0.184	0.186	94.5%				
β_{22}	0.5	0.500	0.436	0.437	94.8%				
σ_1	2	1.986	0.080	0.079	93.9%				
σ_2	4	3.972	0.159	0.159	93.7%				
$\sqrt{D_{11}}$	1	1.000	0.130	0.133	94.9%				
$\sqrt{D_{12}}$	1	0.994	0.088	0.090	93.1%				
ρ_1	0	0.496	0.127	0.132	93.4%				
$\sqrt{D_{21}}$	2	1.995	0.258	0.263	94.9%				
$\sqrt{D_{22}}$	1.5	1.487	0.152	0.159	93.0%				
ρ_2	0.5	0.497	0.138	0.145	92.5%				
R	0.5	0.553	—	—	—				

Table S4: Simulation results for two correlated outcomes, with a random intercept and a random slope on each outcome. The errors were distributed as Weibull.

Work Corr.	True	Joint Model				Separate Models			
		Estimate	SE	Sample SD	Coverage	Estimate	SE	Sample SD	Coverage
Ind.									
β_{10}	1	0.998	0.155	0.154	94.8%	1.004	0.155	0.155	94.8%
β_{11}	1	0.994	0.116	0.114	95.0%	1.004	0.114	0.114	94.7%
β_{12}	3	2.967	0.226	0.227	94.4%	2.998	0.222	0.225	94.9%
β_{20}	1.5	1.479	0.308	0.315	94.0%	1.500	0.305	0.312	94.0%
β_{21}	2	1.959	0.201	0.221	95.8%	1.998	0.185	0.186	95.1%
β_{22}	0.5	0.473	0.435	0.641	93.8%	0.488	0.432	0.446	94.4%
σ_1	2	1.973	0.201	0.222	87.7%	1.984		0.230	
σ_2	4	3.863	0.546	0.660	82.0%	3.923		0.721	
D_{11}	1	0.972	0.156	0.161	95.1%	0.982		0.191	
D_{12}	1	0.979	0.098	0.100	92.6%	0.996		0.100	
ρ_1	0.5	0.507	0.165	0.174	92.7%	0.499		0.183	
D_{21}	2	1.990	0.407	3.211	95.2%	1.949		0.463	
D_{22}	1.5	1.469	0.215	0.442	92.8%	1.504		0.232	
ρ_2	0.5	0.506	0.191	0.233	89.5%	0.495		0.244	
Exch.									
β_{10}	1	0.998	0.154	0.154	94.7%				
β_{11}	1	0.994	0.115	0.114	95.1%				
β_{12}	3	2.965	0.225	0.226	94.2%				
β_{20}	1.5	1.480	0.307	0.308	94.1%				
β_{21}	2	1.956	0.198	0.187	95.9%				
β_{22}	0.5	0.482	0.424	0.447	94.1%				
σ_1	2	1.963	0.202	0.219	87.7%				
σ_2	4	3.846	0.545	0.652	82.2%				
D_{11}	1	0.987	0.143	0.150	95.0%				
D_{12}	1	0.983	0.095	0.097	92.6%				
ρ_1	0.5	0.493	0.146	0.158	93.3%				
D_{21}	2	1.972	0.314	0.585	95.4%				
D_{22}	1.5	1.469	0.195	0.226	93.6%				
ρ_2	0.5	0.495	0.183	0.212	90.9%				
R	0.5	0.510	—	—	—				

Table S5: Simulation results for three correlated outcomes, with a random intercept and slope on each outcome. The errors were normally distributed.

Working Correlation		True Value	Estimate	SE	Sample SD	Coverage
Independent	β_{10}	1	0.999	0.154	0.153	95.0%
	β_{11}	1	0.997	0.114	0.116	94.4%
	β_{12}	3	2.996	0.222	0.222	94.5%
	β_{20}	1.5	1.500	0.154	0.156	94.6%
	β_{21}	2	1.997	0.114	0.115	94.7%
	β_{22}	1.5	1.496	0.222	0.224	94.5%
	β_{30}	0.5	0.502	0.303	0.303	95.0%
	β_{31}	1.5	1.495	0.144	0.147	94.6%
	β_{32}	1	1.002	0.433	0.433	94.6%
	σ_1	2	1.996	0.079	0.082	93.6%
	σ_2	2	1.996	0.079	0.080	94.3%
	σ_3	4	3.992	0.156	0.160	93.3%
	D_{11}	1	0.979	0.142	0.147	94.1%
	ρ_1	0.5	0.508	0.139	0.145	92.6%
	D_{12}	1	0.989	0.090	0.091	93.1%
	D_{21}	1	0.976	0.141	0.144	94.8%
	ρ_2	0.5	0.510	0.139	0.143	93.5%
	D_{22}	1	0.988	0.090	0.093	92.6%
	D_{31}	2	1.963	0.274	0.275	95.3%
	ρ_3	0.3	0.309	0.190	0.199	92.2%
	D_{32}	1	0.977	0.143	0.150	94.0%
Exchangeable	β_{10}	1	0.999	0.154	0.153	95.1%
	β_{11}	1	0.997	0.114	0.115	94.4%
	β_{12}	3	2.996	0.221	0.221	94.5%
	β_{20}	1.5	1.500	0.154	0.156	94.6%
	β_{21}	2	1.997	0.114	0.115	94.4%
	β_{22}	1.5	1.497	0.220	0.223	94.4%
	β_{30}	0.5	0.502	0.303	0.303	94.9%
	β_{31}	1.5	1.495	0.144	0.147	94.6%
	β_{32}	1	1.002	0.433	0.433	94.7%
	σ_1	2	1.985	0.082	0.083	93.2%
	σ_2	2	2.010	0.082	0.080	94.9%
	σ_3	4	3.940	0.158	0.161	91.2%
	D_{11}	1	0.989	0.134	0.139	94.1%
	ρ_1	0.5	0.500	0.132	0.137	93.0%
	D_{12}	1	0.997	0.089	0.090	93.8%
	D_{21}	1	0.946	0.135	0.138	93.4%
	ρ_2	0.5	0.531	0.134	0.138	91.9%
	D_{22}	1	0.985	0.088	0.091	92.4%
	D_{31}	2	2.057	0.259	0.263	94.2%
	ρ_3	0.3	0.272	0.178	0.184	92.7%
	D_{32}	1	1.006	0.141	0.147	94.3%
	R	0.3	0.339	—	—	—

Working Correlation		True Value	Estimate	SE	Sample SD	Coverage
Unstructured	β_{10}	1	0.999	0.154	0.153	95.1%
	β_{11}	1	0.997	0.114	0.115	94.2%
	β_{12}	3	2.996	0.220	0.221	94.3%
	β_{20}	1.5	1.499	0.154	0.158	94.7%
	β_{21}	2	1.996	0.114	0.119	94.6%
	β_{22}	1.5	1.496	0.220	0.223	94.3%
	β_{30}	0.5	0.502	0.303	0.303	95.0%
	β_{31}	1.5	1.495	0.144	0.146	94.6%
	β_{32}	1	1.002	0.432	0.432	94.6%
	σ_1	2	1.989	0.081	0.081	94.0%
	σ_2	2	1.985	0.082	0.079	94.6%
	σ_3	4	3.983	0.157	0.160	93.5%
	D_{11}	1	0.993	0.131	0.135	94.7%
	ρ_1	0.5	0.497	0.129	0.134	93.2%
	D_{12}	1	0.991	0.088	0.089	93.7%
	D_{21}	1	0.999	0.130	0.170	95.0%
	ρ_2	0.5	0.493	0.128	0.128	93.7%
	D_{22}	1	0.993	0.088	0.091	93.1%
	D_{31}	2	1.978	0.265	0.269	94.9%
	ρ_3	0.3	0.302	0.185	0.194	92.6%
	D_{32}	1	0.982	0.142	0.148	93.9%
	R_{21}	0.5	0.537	—	—	—
	R_{31}	0.1	0.122	—	—	—
	R_{32}	0.3	0.341	—	—	—

Table S6: Simulation results for three correlated outcomes, with a random intercept and slope on each outcome. The errors were distributed as mixture of normals.

Working Correlation		True Value	Estimate	SE	Sample SD	Coverage
Independent	β_{10}	1	1.006	0.155	0.156	94.4%
	β_{11}	1	0.998	0.114	0.116	93.6%
	β_{12}	3	2.988	0.223	0.224	94.2%
	β_{20}	1.5	1.502	0.154	0.157	94.6%
	β_{21}	2	1.999	0.114	0.112	94.9%
	β_{22}	1.5	1.494	0.222	0.227	94.4%
	β_{30}	0.5	0.508	0.303	0.308	94.5%
	β_{31}	1.5	1.501	0.143	0.146	94.5%
	β_{32}	1	0.991	0.431	0.438	94.4%
	σ_1	2	1.999	0.077	0.079	93.9%
	σ_2	2	1.995	0.077	0.078	94.1%
	σ_3	4	3.989	0.155	0.157	94.3%
	D_{11}	1	0.977	0.142	0.144	94.8%
	ρ_1	0.5	0.513	0.138	0.144	92.5%
	D_{12}	1	0.988	0.090	0.091	93.5%
	D_{21}	1	0.977	0.142	0.146	94.4%
	ρ_2	0.5	0.508	0.139	0.144	92.9%
	D_{22}	1	0.989	0.090	0.092	93.0%
	D_{31}	2	1.959	0.273	0.276	95.0%
	ρ_3	0.3	0.312	0.191	0.199	92.8%
	D_{32}	1	0.974	0.142	0.153	93.3%
Exchangeable	β_{10}	1	1.006	0.154	0.156	94.3%
	β_{11}	1	0.997	0.114	0.116	93.7%
	β_{12}	3	2.987	0.222	0.223	94.3%
	β_{20}	1.5	1.502	0.154	0.156	94.5%
	β_{21}	2	1.999	0.114	0.112	94.8%
	β_{22}	1.5	1.493	0.221	0.225	94.2%
	β_{30}	0.5	0.508	0.303	0.308	94.4%
	β_{31}	1.5	1.501	0.144	0.146	94.7%
	β_{32}	1	0.990	0.432	0.438	94.2%
	σ_1	2	1.987	0.080	0.080	93.8%
	σ_2	2	2.009	0.080	0.079	95.1%
	σ_3	4	3.937	0.158	0.158	91.8%
	D_{11}	1	0.988	0.134	0.136	94.7%
	ρ_1	0.5	0.505	0.131	0.136	93.2%
	D_{12}	1	0.996	0.089	0.090	94.0%
	D_{21}	1	0.948	0.136	0.141	92.8%
	ρ_2	0.5	0.528	0.135	0.141	91.6%
	D_{22}	1	0.985	0.088	0.090	92.8%
	D_{31}	2	2.052	0.258	0.264	94.5%
	ρ_3	0.3	0.275	0.178	0.185	92.9%
	D_{32}	1	1.004	0.141	0.151	93.8%
	R	0.3	0.340	—	—	—

Working Correlation		True Value	Estimate	SE	Sample SD	Coverage
Unstructured	β_{10}	1	1.006	0.154	0.156	94.5%
	β_{11}	1	0.998	0.113	0.115	93.8%
	β_{12}	3	2.988	0.222	0.222	94.3%
	β_{20}	1.5	1.502	0.154	0.156	94.5%
	β_{21}	2	1.999	0.114	0.112	94.9%
	β_{22}	1.5	1.493	0.220	0.224	94.4%
	β_{30}	0.5	0.508	0.303	0.308	94.4%
	β_{31}	1.5	1.501	0.143	0.146	94.6%
	β_{32}	1	0.991	0.431	0.438	94.3%
	σ_1	2	1.992	0.079	0.078	94.4%
	σ_2	2	1.984	0.081	0.078	94.5%
	σ_3	4	3.980	0.157	0.155	94.1%
	D_{11}	1	0.991	0.131	0.132	94.7%
	ρ_1	0.5	0.501	0.128	0.133	93.0%
	D_{12}	1	0.990	0.088	0.090	93.5%
	D_{21}	1	0.999	0.127	0.132	94.4%
	ρ_2	0.5	0.490	0.125	0.130	93.0%
	D_{22}	1	0.994	0.087	0.090	93.4%
	D_{31}	2	1.974	0.264	0.267	95.2%
	ρ_3	0.3	0.303	0.186	0.193	92.7%
	D_{32}	1	0.979	0.141	0.151	93.3%
	R_{21}	0.5	0.537	—	—	—
	R_{31}	0.1	0.121	—	—	—
	R_{32}	0.3	0.342	—	—	—

Table S7: Simulation results ($N = 1000$) for two correlated outcomes, with a random intercept on each outcome. The errors were normally distributed.

Working Correlation		True Value	Estimate	SE	Sample SD	Coverage
Independent	β_{10}	1	1.000	0.047	0.045	95.8%
	β_{11}	1	1.000	0.015	0.014	96.6%
	β_{12}	3	3.004	0.065	0.063	96.4%
	β_{20}	1.5	1.503	0.047	0.049	94.0%
	β_{21}	2	2.000	0.015	0.015	95.0%
	β_{22}	0.5	0.501	0.065	0.067	94.6%
	σ_1	2	1.998	0.022	0.023	93.4%
	σ_2	2	1.999	0.022	0.021	95.8%
	$\sqrt{D_1}$	1	0.999	0.041	0.043	93.4%
	$\sqrt{D_2}$	1	0.998	0.041	0.042	94.4%
Exchangeable	β_{10}	1	1.000	0.047	0.045	95.8%
	β_{11}	1	1.000	0.015	0.014	96.6%
	β_{12}	3	3.004	0.065	0.063	96.0%
	β_{20}	1.5	1.503	0.047	0.049	93.8%
	β_{21}	2	2.000	0.015	0.015	95.0%
	β_{22}	0.5	0.501	0.065	0.067	94.6%
	σ_1	2	1.997	0.022	0.023	93.6%
	σ_2	2	1.999	0.022	0.021	96.0%
	$\sqrt{D_1}$	1	1.001	0.040	0.042	92.6%
	$\sqrt{D_2}$	1	1.000	0.040	0.040	93.8%
	R	0.3	0.317	—	—	—

Table S8: Simulation results ($N = 1000$) for two correlated outcomes, with a random intercept on each outcome. The errors were distributed as mixture of normals.

Working Correlation		True Value	Estimate	SE	Sample SD	Coverage
Independent	β_{10}	1	0.998	0.047	0.045	95.4%
	β_{11}	1	0.999	0.015	0.015	93.6%
	β_{12}	3	3.000	0.065	0.065	94.4%
	β_{20}	1.5	1.504	0.047	0.046	94.8%
	β_{21}	2	1.999	0.015	0.015	96.0%
	β_{22}	0.5	0.499	0.065	0.065	95.4%
	σ_1	2	2.000	0.022	0.022	96.2%
	σ_2	2	2.001	0.022	0.023	95.2%
	$\sqrt{D_1}$	1	0.998	0.041	0.038	96.4%
	$\sqrt{D_2}$	1	0.999	0.041	0.043	93.2%
Exchangeable	β_{10}	1	0.998	0.047	0.045	95.4%
	β_{11}	1	0.999	0.015	0.015	93.8%
	β_{12}	3	3.000	0.065	0.065	93.6%
	β_{20}	1.5	1.504	0.047	0.046	94.8%
	β_{21}	2	1.999	0.015	0.014	96.0%
	β_{22}	0.5	0.499	0.065	0.065	95.4%
	σ_1	2	2.000	0.022	0.022	96.0%
	σ_2	2	2.001	0.022	0.023	95.4%
	$\sqrt{D_1}$	1	0.999	0.040	0.038	96.0%
	$\sqrt{D_2}$	1	1.001	0.040	0.041	93.8%
R		0.3	0.318	—	—	—

Table S9: Simulation results ($N = 1000$) for two correlated outcomes, with a random intercept and a random slope on each outcome. The errors were normally distributed.

Working Correlation		True Value	Estimate	SE	Sample SD	Coverage
Independent	β_{10}	1	1.000	0.049	0.050	94.4%
	β_{11}	1	1.001	0.036	0.036	94.4%
	β_{12}	3	2.996	0.071	0.072	95.4%
	β_{20}	1.5	1.505	0.097	0.102	93.6%
	β_{21}	2	1.995	0.058	0.062	92.0%
	β_{22}	0.5	0.495	0.140	0.135	95.2%
	σ_1	2	1.999	0.025	0.024	94.8%
	σ_2	4	4.001	0.050	0.051	95.0%
	$\sqrt{D_{11}}$	1	0.996	0.045	0.044	96.2%
	ρ_1	0.5	0.501	0.043	0.042	95.0%
	$\sqrt{D_{12}}$	1	1.000	0.029	0.030	94.4%
	$\sqrt{D_{21}}$	2	1.995	0.089	0.091	95.0%
	ρ_2	0.5	0.503	0.046	0.047	94.4%
	$\sqrt{D_{22}}$	1.5	1.497	0.050	0.051	93.4%
Exchangeable						
	β_{10}	1	1.001	0.049	0.049	94.2%
	β_{11}	1	1.001	0.036	0.036	93.8%
	β_{12}	3	2.996	0.070	0.071	95.2%
	β_{20}	1.5	1.504	0.097	0.102	93.4%
	β_{21}	2	1.995	0.058	0.062	92.2%
	β_{22}	0.5	0.495	0.139	0.136	95.6%
	σ_1	2	1.990	0.026	0.023	95.2%
	σ_2	4	3.985	0.051	0.049	94.8%
	$\sqrt{D_{11}}$	1	1.013	0.042	0.042	94.2%
	ρ_1	0.5	0.489	0.040	0.040	94.8%
	$\sqrt{D_{12}}$	1	1.004	0.028	0.029	95.4%
	$\sqrt{D_{21}}$	2	2.025	0.082	0.083	93.6%
	ρ_2	0.5	0.491	0.043	0.044	94.0%
	$\sqrt{D_{22}}$	1.5	1.503	0.049	0.049	94.8%
	R	0.5	0.549	—	—	—

Table S10: Simulation results ($N = 1000$) for two correlated outcomes, with a random intercept and a random slope on each outcome. The errors were distributed as mixture of normals.

Working Correlation		True Value	Estimate	SE	Sample SD	Coverage
Independent	β_{10}	1	1.000	0.049	0.051	93.6%
	β_{11}	1	0.997	0.036	0.036	96.2%
	β_{12}	3	3.002	0.071	0.072	94.4%
	β_{20}	1.5	1.504	0.097	0.099	95.0%
	β_{21}	2	1.999	0.058	0.057	95.6%
	β_{22}	0.5	0.499	0.140	0.146	93.0%
	σ_1	2	2.000	0.025	0.026	94.6%
	σ_2	4	3.996	0.050	0.052	93.2%
	$\sqrt{D_{11}}$	1	0.999	0.045	0.044	96.2%
	ρ_1	0.5	0.503	0.043	0.043	95.4%
	$\sqrt{D_{12}}$	1	1.000	0.029	0.029	95.6%
	$\sqrt{D_{21}}$	2	2.002	0.089	0.092	94.6%
	ρ_2	0.5	0.501	0.046	0.048	93.2%
	$\sqrt{D_{22}}$	1.5	1.496	0.050	0.051	95.2%
Exchangeable	β_{10}	1	0.999	0.049	0.052	93.0%
	β_{11}	1	0.997	0.036	0.035	96.4%
	β_{12}	3	3.002	0.070	0.071	94.8%
	β_{20}	1.5	1.504	0.097	0.098	94.8%
	β_{21}	2	1.999	0.058	0.057	95.4%
	β_{22}	0.5	0.499	0.139	0.144	93.4%
	σ_1	2	1.990	0.026	0.026	91.8%
	σ_2	4	3.980	0.051	0.051	92.2%
	$\sqrt{D_{11}}$	1	1.015	0.042	0.042	93.8%
	ρ_1	0.5	0.492	0.040	0.040	94.6%
	$\sqrt{D_{12}}$	1	1.004	0.028	0.028	94.8%
	$\sqrt{D_{21}}$	2	2.031	0.082	0.086	92.8%
	ρ_2	0.5	0.490	0.043	0.044	93.8%
	$\sqrt{D_{22}}$	1.5	1.503	0.049	0.050	94.2%
	R	0.5	0.550	—	—	—

Table S11: Comparison of two intervention groups while adjusting for baseline covariates (separate models).

	Fat			Fiber			Veg.		
	Estimate	SE	p-value	Estimate	SE	p-value	Estimate	SE	p-value
Intercept	28.114	1.815	<0.001	14.355	0.786	<0.001	1.828	0.170	<0.001
Weight0	0.005	0.020	0.781	-0.035	0.008	<0.001	-0.003	0.002	0.074
Fat0	0.386	0.015	<0.001	-0.036	0.007	<0.001	-0.007	0.001	<0.001
Fib0	0.094	0.047	0.045	0.483	0.020	<0.001	0.034	0.004	<0.001
Veg0	-0.173	0.227	0.448	0.526	0.099	<0.001	0.392	0.021	<0.001
Time	5.215	0.417	<0.001	-2.860	0.156	<0.001	-0.249	0.038	<0.001
Intervention	-9.821	0.625	<0.001	6.978	0.271	<0.001	1.490	0.059	<0.001
σ	14.073	-	-	5.259	-	-	1.289	-	-
\sqrt{D}	11.063	-	-	5.272	-	-	1.060	-	-

Table S12: Time trajectory from baseline to 1 and 4 years (phone counseling group, separate models).

	Fat			Fiber			Veg.		
	Estimate	SE	p-value	Estimate	SE	p-value	Estimate	SE	p-value
Intercept	55.671	0.582	<0.001	21.553	0.285	<0.001	2.995	0.060	<0.001
Year 1	-14.468	0.633	<0.001	7.765	0.274	<0.001	1.721	0.061	<0.001
Year 4	-8.268	0.633	<0.001	3.869	0.274	<0.001	1.283	0.061	<0.001
σ	14.775	-	-	6.404	-	-	1.420	-	-
\sqrt{D}	12.276	-	-	6.878	-	-	1.371	-	-

Table S13: Time trajectory from baseline to 1 and 4 years (active control group, separate models).

	Fat			Fiber			Veg.		
	Estimate	SE	p-value	Estimate	SE	p-value	Estimate	SE	p-value
Intercept	56.530	0.623	<0.001	21.385	0.230	<0.001	2.945	0.049	<0.001
Year 1	-4.233	0.640	<0.001	-0.193	0.204	0.346	0.067	0.050	0.182
Year 4	0.075	0.640	0.906	-2.099	0.204	<0.001	-0.007	0.050	0.889
σ	15.582	-	-	4.971	-	-	1.221	-	-
\sqrt{D}	14.726	-	-	6.142	-	-	1.171	-	-

Table S14: Comparison of two intervention groups with interaction by time.

	Fat			Fiber			Veg.		
	Estimate	SE	p-value	Estimate	SE	p-value	Estimate	SE	p-value
Intercept	26.980	1.796	<0.001	14.372	0.772	<0.001	1.757	0.167	<0.001
Weight0	0.008	0.019	0.690	-0.038	0.008	<0.001	-0.004	0.002	0.023
Fat0	0.377	0.016	<0.001	-0.034	0.006	<0.001	-0.006	0.001	<0.001
Fib0	0.099	0.044	0.025	0.490	0.023	<0.001	0.034	0.005	<0.001
Veg0	-0.094	0.226	0.677	0.555	0.100	<0.001	0.390	0.023	<0.001
Time	1.434	0.202	<0.001	-0.621	0.063	<0.001	-0.024	0.016	0.126
Intervention	-11.287	0.823	<0.001	8.352	0.366	<0.001	1.784	0.080	<0.001
Time × Int.	0.698	0.271	0.010	-0.670	0.103	<0.001	-0.125	0.025	<0.001
σ	14.104	0.080	-	6.138	0.016	-	1.474	0.025	-
D	10.985	0.290	-	4.143	0.120	-	0.754	0.030	-
R				Fat	Fiber	Veg.			
				Fat	1				
				Fiber	-0.160	1			
				Veg.	-0.060	0.705	1		

Table S15: Time trajectory from baseline to 1 and 4 years.

	Fat			Fiber			Veg.		
	Estimate	SE	p-value	Estimate	SE	p-value	Estimate	SE	p-value
Intercept	56.299	0.560	<0.001	21.141	0.204	<0.001	2.876	0.043	<0.001
Year 1	-4.302	0.582	<0.001	-0.092	0.194	0.637	0.096	0.045	0.035
Year 4	0.000	0.646	1.000	-1.870	0.203	<0.001	0.037	0.050	0.462
Intervention	-0.228	0.788	0.772	-0.067	0.294	0.820	0.060	0.061	0.329
Year 1 × Int.	-10.419	0.823	<0.001	7.862	0.331	<0.001	1.647	0.072	<0.001
Year 4 × Int.	-8.299	0.910	<0.001	5.808	0.337	<0.001	1.260	0.079	<0.001
σ	15.425	0.823	-	6.982	0.788	-	1.566	0.910	-
D	13.639	0.045	-	4.668	0.043	-	0.858	0.050	-
R				Fat	Fiber	Veg.			
				Fat	1				
				Fiber	0.002	1			
				Veg.	0.063	0.661	1		